

Name of the Course	: B.A. (Prog)
Unique Paper Code	: 62351201_OC
Name of the Paper	: Algebra
Semester	: II
Duration	: 3 Hours
Maximum Marks	: 75

Attempt any four questions. All questions carry equal marks.

1. Consider the vector space  $\mathbb{R}^3$  and its subset  $S$

$$S = \{(a, b, c) : 3a - 4b + c = 0, a + 2b - c = 0, a, b, c \in \mathbb{R}\}$$

Show that  $S$  is a subset of  $\mathbb{R}^3$  and also find  $\dim S$ .

Determine whether or not the vectors  $(1, -3, 2)$ ,  $(2, 4, 1)$  and  $(1, 1, 1)$  form a basis of  $\mathbb{R}^3$

Are the following sets linearly independent or linearly dependent?

$$A = \{(3 - i, 2 + 2i, 4), (2, 2 + 4i, 3), (1 - i, -2i, 1)\}; \quad B = \{(1, 2, 3), (1, 3, 2), (3, 7, 8)\}$$

2. Solve the system of equations:

$$3x + 4y - 6z + w = 7$$

$$x - 2y + 3z - 2w = -1$$

$$x - 3y + 4z - w = -2$$

$$5x - y + z - 2w = 4$$

Find the rank of the matrix

$$A = \begin{pmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$

Verify that the matrix  $B$  satisfies its own characteristic equation and hence find its inverse

$$B = \begin{pmatrix} 12 & -1 & 4 \\ -1 & 12 & -1 \\ 4 & -1 & 12 \end{pmatrix}.$$

3. Show that

$$2^7 \sin^4 2\theta \sin^4 \theta = \cos 12\theta - 4 \cos 10\theta + 2 \cos 8\theta + 12 \cos 6\theta - 17 \cos 4\theta - 8 \cos 2\theta + 14$$

Sum the series

$$\sin \theta \sin 2\theta + \sin 2\theta \sin 3\theta + \sin 3\theta \sin 4\theta + \dots \text{ upto } n \text{ terms, } \quad n \in \mathbb{N}$$

Solve the equation

$$(z + 1)^7 = z^7 [\cos 7\alpha + i \sin 7\alpha].$$

4. Solve the equation  $x^3 - 9x^2 + 23x - 15 = 0$  ; the roots being in A.P.

If  $\alpha, \beta, \gamma$  are roots of the equation  $x^3 - mx^2 + nx - s = 0$ , find the value of

$$\sum \alpha^2 \beta, \quad \sum \frac{\alpha^2}{\beta \gamma}$$

Find all the roots of the equation  $x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$ , where it is given that  $2 + \sqrt{3}$  is one of the roots.

5. Find the multiplicative inverses of the given elements if they exist:

$$[11]_{16} \text{ in } \mathbb{Z}_{16} \quad \text{and} \quad [38]_{83} \text{ in } \mathbb{Z}_{83}.$$

Let  $a$  be a fixed element of a group  $G$  and let

$$N(t) = \{x \in G : xt = tx\}.$$

Show that  $N(t)$  is a subgroup of  $G$ .

Find the order and inverse of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 4 & 3 & 6 & 1 \end{pmatrix}.$$

6. In permutation group  $S_6$ , if  $\alpha = (2 \ 3 \ 4)$  and  $\beta = (2 \ 5 \ 6)$ , then find a permutation  $\phi$  such that  $\phi \alpha \phi^{-1} = \beta$ .

Let  $a$  be a fixed element of a ring  $R$ , and let

$$I_s = \{x \in R : sx = 0\}.$$

Show that  $I_s$  is a subring of  $R$ .

Show that  $\mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$  is a commutative ring w.r.t. addition modulo 8 and multiplication modulo 8.